Unit 1: Limits and Continuity

Content Area: **Math**
Course(s): **CALCULUS HONORS, AP CALC**
Time Period: **Marking Period 1**
Length: **5 Weeks**
Status: **Published**

Standards

Math Standards

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write □(□)/□(□) in the form □(□) + □(□)/□(□), where □(□), □(□), □(□), and □(□) are polynomials with the degree of □(□) less than the degree of □(□), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Mathematical Practices

- 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression.
- 1.E Apply appropriate mathematical rules or procedures, with and without technology.
- 2.B Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
- 2.C Identify a re-expression of mathematical information presented in a given representation.
- 2.D Identify how mathematical characteristics or properties of functions are related in different representations.
- 3.B Identify an appropriate mathematical definition, theorem, pr test to apply.
- 3.C Confirm whether hypotheses or conditions of a selected definition,, theorem, or test have been satisfied.
- 3.D Apply an appropriate mathematical definition, theorem, or test.
- 3.E Provide reasons or rationales for solutions or conclusions.

Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).

TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Transfer Goals

Transfer Goals

Limits introduce the subtle distinction between evaluating a function at a point and considering what value the function is approaching, if any, as x approaches a point.

Concepts

Essential Questions

- Can change occur at an instant?
- How does knowing the value of a limit, or that a limit does not exist, help you to make sense of interesting features of functions and their graphs?
- How do we close loopholes so that a conclusion about a function is always true?

Understandings

- Calculus allows us to generalize knowledge about motion to diverse problems involving change.
- Reasoning with definitions, theorems, and properties can be used to justify claims about limits.
- Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.
- Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

Critical Knowledge and Skills

Knowledge

Students will know:

- Calculus uses limits to understand and model dynamic change.
- Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero.
- The limit concept allows us to define instantaneous rate of change in terms of average rates of change.
- Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim f(x) = R$.
- A limit can be expressed in multiple ways, including graphically, numerically, and analytically.
- The concept of a limit includes one sided limits.
- Graphical information about a function can be used to estimate limits.
- Because of issues of scale, graphical representations of functions may miss important function behavior.
- A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- Numerical information can be used to estimate limits.
- One-sided limits can be determined analytically or graphically.
- Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems.
- It may be necessary or helpful to rearrange expressions into equivalent forms before evaluating limits.
- The limit of a function may be found by using the squeeze theorem.
- Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim f(x)$ exists, and $\lim f(x) = f(c)$.
- A function is continuous on an interval if the function is continuous at each point in the interval.
- Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains.
- If the limit of a function exists at a discontinuity in its graph, then it is possible to remove the discontinuity by defining or redefining the value of the function at that point, so it equals the value of the limit of the function as x approaches that point.
- In order for a piecewise-defined function to be continuous at a boundary to the partition of its domain, the value of the expression defining the function on one side of the boundary must equal the value of the expression defining the other side of the boundary, as well as the value of the function at the boundary.
- The concept of a limit can be extended to include infinite limits.
- Asymptotic and unbounded behavior of functions can be described and explained using limits.
- The concept of a limit can be extended to include limits at infinity.
- Limits at infinity describe end behavior.
- Relative magnitudes of functions and their rates of change can be compared using limits.
- If f is a continuous function on the closed interval $[a, b]$ and d is a number between $f(a)$ and $f(b)$, then the Intermediate Value Theorem guarantees that there is at least one number c between a and b, such that $f(c) = d$.

Skills

Students will be able to:

- Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.
- Represent limits analytically using correct notation.
- Interpret limits expressed in analytic notation.
- Estimate limits of functions.
- Determine the limits of functions using limit theorems.
- Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.
- Justify conclusions about continuity at a point using the definition.
- Determine intervals over which a function is continuous.
- Determine values of x or solve for parameters that make discontinuous functions continuous, if possible.
- Interpret the behavior of functions using limits involving infinity
- Explain the behavior of a function on an interval using the Intermediate Value Theorem.

Assessment and Resources

School Formative Assessment Plan (Other Evidence)

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

School Summative Assessment Plan

- Unit Assessment

Primary Resources

- Sullivan, Michael, and Kathleen Miranda. *Calculus: Early Transcendentals*. W.H. Freeman and Company, 2014.

- Finney, Ross L. *Calculus: Graphical, Numerical, Algebraic*. Pearson Prentice Hall, 2007.

Supplementary Resources

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMP
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- Rogawski, Jon. *Calculus. Early Transcendentals*. Freeman, 2007.
- Dietiker, Leslie, et al. *College Preparatory Mathematics: Calculus*. CPM Educational Program, 2003.

Technology Integration and Differentiated Instruction

Technology Integration

Google Products

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
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One to One Student's Laptop

All students within the West Deptford School District are given a Chromebook, allowing for 21st century learning to occur within every lesson/topic.

Graphing Calculators

All students all required to utilize a graphing calculator, allowing for 21st century learning to occur within

every lesson/topic.

Additional Support Videos

Videos can be assigned from PatrickJMP, Kahn Academy, YouTube, etc. to support each of the lessons within this topic.

Differentiated Instruction

Gifted Students (N.J.A.C.6A:8-3.1)

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

English Language Learners (N.J.A.C.6A:15)

- When discussing different ways of solving problems, focus on keywords and phrases. ELL Students might also benefit by using concrete objects to demonstrate different concepts.
- Create place cards or simple signs for students using the vocabulary words. They can use the signs as a reference throughout the lesson.
- Work with ELL Teacher to allow for all assignments to be completed with extra time.
- Pair ELL students with a student who is fluent in English.

At-Risk Students (N.J.A.C.6A:8-4.3c)

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

Special Education Students (N.J.A.C.6A:8-3.1)

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- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

Interdisciplinary Connections

ELA - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

SCIENCE - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

SOCIAL STUDIES - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

WORLD LANGUAGES -

VISUAL/PERFORMING ARTS -

APPLIED TECHNOLOGY - Students will make use of graphical interfaces such as Desmos and the TI graphing calculator suite.

BUSINESS EDUCATION -

GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- Introduction to Calculus
- Defining Limits and Using Limit Notation

Week 2:

- Estimating Limit Values from Graphs
- Estimating Limit Values from Tables
- Determining Limits Using Algebraic Properties of Limits
- Determining Limits Using Algebraic Manipulation

Week 3:

- Selecting Procedures for Determining Limits
- Determining Limits Using the Squeeze Theorem
- Connecting Multiple Representations of Limits (Note: Use this lesson as a review for determining limits.)
- Determining Limits Quiz

Week 4:

- Exploring Types of Discontinuities
- Defining Continuity at a Point
- Confirming Continuity over an Interval

- Removing Discontinuities

Week 5:

- Connecting Infinite Limits and Vertical Asymptotes
- Connecting Limits at Infinity and Horizontal Asymptotes
- Working with the Intermediate Value Theorem
- Personal Progress Check 1
- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Honors Calculus

Week 1:

- Section 1.1 Limits of Functions Using Numerical and Graphical Techniques

Week 2:

- Section 1.2 Limits of Functions Using Properties of Limits
- Section 1.2 Review
- Section 1.2 Quiz

Week 3:

- Section 1.3 Continuity

Week 4:

- Section 1.5 Infinite Limits; Limits at Infinity; Asymptotes
- Section 1.5 Review
- Section 1.5 Quiz

Week 5:

- Unit Review
- Unit Test

Unit 2: Differentiation - Definition and Fundamental Properties

Content Area: **Math**
Course(s): **CALCULUS HONORS, AP CALC**
Time Period: **Marking Period 1**
Length: **4 Weeks**
Status: **Published**

Standards

Math Standards

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – □, π + □, and 2π – □ in terms of their values for □, where □ is any real number.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.

Mathematical Practices

- 1.D: Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.
- 1.E: Apply appropriate mathematical rules or procedures, with and without technology.
- 2.B: Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
- 3.E: Provide reasons or rationales for solutions and conclusions.
- 4.C: Use appropriate mathematical symbols and notation.

Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility

for accomplishing a specified task (e.g., W.11-12.6.).

Transfer Goals

Transfer Goals

Derivatives allow us to determine instantaneous rates of change.

Concepts

Essential Questions

- How can a state determine the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) based on a model for the number of graduates as a function of the state's education budget?
- Why do mathematical properties and rules for simplifying and evaluating limits apply to differentiation?
- If you knew that the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) was a positive number, what might that tell you about the number of graduates at that level of investment?

Understandings

- Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.
- Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both.
- Recognizing opportunities to apply derivative rules can simplify differentiation.
- Reasoning with definitions, theorems, and properties can be used to determine a limit.

Critical Knowledge and Skills

Knowledge

Students will know:

- The difference quotients express the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at $x = a$ can be expressed by the limit of the difference quotients, provided the limit exists. These are equivalent forms of the definition of the derivative and are denoted $f'(a)$.
- The derivative of f is the function whose value at x is the limit of the difference quotient, provided the limit exists.
- For $y = f(x)$, notations for the derivative include dy/dx , $f'(x)$, and y'.
- The derivative can be represented graphically, numerically, analytically, and verbally
- The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point.
- The derivative at a point can be estimated from information given in tables or graphs.
- Technology can be used to calculate or estimate the value of a derivative of a function at a point.
- If a function is differentiable at a point, then it is continuous at that point. In particular, if a point is not in the domain of f, then it is not in the domain of f'.
- A continuous function may fail to be differentiable at a point in its domain.
- Direct application of the definition of the derivative and specific rules can be used to calculate the derivative for functions of the form $f(x) = x'$.
- Sums, differences, and constant multiples of functions can be differentiated using derivative rules.
- The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions.
- Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions.
- In some cases, recognizing an expression for the definition of the derivative of a function whose derivative is known offers a strategy for determining a limit.
- Derivatives of products of differentiable functions can be found using the product rule.
- Derivatives of quotients of differentiable functions can be found using the quotient rule.
- Rearranging tangent, cotangent, secant, and cosecant functions using identities allows differentiation using derivative rules.

Skills

Students will be able to:

- Determine average rates of change using difference quotients.
- Represent the derivative of a function as the limit of a difference quotient.
- Determine the equation of a line tangent to a curve at a given point.
- Estimate derivatives.
- Explain the relationship between differentiability and continuity.
- Calculate derivatives of familiar functions.
- Interpret a limit as a definition of a derivative.
- Calculate derivatives of products and quotients of differentiable functions.

Assessment and Resources

School Formative Assessment Plan (Other Evidence)

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

School Summative Assessment Plan

- Unit Assessment

Primary Resources

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Additional Support Videos

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Differentiated Instruction

Gifted Students (N.J.A.C.6A:8-3.1)

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English Language Learners (N.J.A.C.6A:15)

- When discussing different ways of solving problems, focus on keywords and phrases. ELL Students might also benefit by using concrete objects to demonstrate different concepts.
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WORLD LANGUAGES -

VISUAL/PERFORMING ARTS -

APPLIED TECHNOLOGY -

BUSINESS EDUCATION -

GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- Defining Average and Instantaneous Rate of Change at a Point
- Defining the Derivative of a Function and Using Derivative Notation
- Estimating Derivatives of a Function at a Point

Week 2:

- Connecting Differentiability and Continuity - Determining When Derivatives Do and Do Not Exist
- Definition of the Derivative Review and Quiz
- Applying the Power Rule

Week 3:

- Derivative Rules: Constant, Sum, Difference, and Constant Multiple
- Derivatives of $\sin x$, $\cos x$, e^x, and $\ln(x)$.
- The Product Rule

Week 4:

- The Quotient Rule
- Finding the Derivatives of $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$.
- Personal Progress Check 2
- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Honors Calculus

Week 1:

- Section 2.1 Rates of Change and the Derivative
- Section 2.1 Review
- Section 2.1 Quiz

Week 2:

- Section 2.3 The Derivative of a Polynomial Function; The Derivative of $y= e^x$
- Section 2.4 Differentiating the Product and the Quotient of Two Functions; Higher-Order Derivatives

Week 3:

- Section 2.4 Review
- Section 2.4 Quiz
- Section 2.5 The Derivative of the Trigonometric Functions

Week 4:

- Section 2.5 Review
- Section 2.5 Quiz
- Unit Review
- Unit Test

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Content Area: **Math**
Course(s): **CALCULUS HONORS, AP CALC**
Time Period: **Marking Period 2**
Length: **3 Weeks**
Status: **Published**

Standards

Math Standards

MA.F-BF.B.4	Find inverse functions.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - \square$, $\pi + \square$, and $2\pi - \square$ in terms of their values for $\square$, where $\square$ is any real number.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Mathematical Practices

- 1.C: Identify an appropriate mathematical rule or procedure based on the classification of a given expression.
- 1.E: Apply appropriate mathematical rules or procedures, with and without technology.
- 3.G: Confirm that solutions are accurate and appropriate.

Life Literacies and Key Skills

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Transfer Goals

Transfer Goals

In this unit, students learn how to differentiate composite functions using the chain rule and apply that understanding to determine derivatives of implicit and inverse functions.

Concepts

Essential Questions

- If pressure experienced by a diver is a function of depth and depth is a function of time, how might we find the rate of change in pressure with respect to time?

Understandings

- Recognizing opportunities to apply derivative rules can simplify differentiation.

Critical Knowledge and Skills

Knowledge

Students will know:

- The chain rule provides a way to differentiate composite functions.
- The chain rule is the basis for implicit differentiation.
- The chain rule and definition of an inverse function can be used to find the derivative of an inverse function, provided the derivative exists.
- The chain rule applied with the definition of an inverse function, or the formula for the derivative of an

inverse function, can be used to find the derivatives of inverse trigonometric functions.
- Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher-order derivatives of f.
- Higher-order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include d^2y/dx^2, $f''(x)$, and y''. Higher-order derivatives can be denoted d^ny/dx^n or $f^{(n)}(x)$.

Skills

Students will be able to:

- Calculate derivatives of compositions of differentiable functions.
- Calculate derivatives of implicitly defined functions.
- Calculate derivatives of inverse and inverse trigonometric functions.
- Determine higher order derivatives of a function.

Assessment and Resources

School Formative Assessment Plan (Other Evidence)

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

School Summative Assessment Plan

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Interdisciplinary Connections

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WORLD LANGUAGES -

VISUAL/PERFORMING ARTS -

APPLIED TECHNOLOGY -

BUSINESS EDUCATION -

GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- The Chain Rule
- Implicit Differentiation
- Differentiating Inverse Functions

Week 2:

- Differentiating Inverse Trigonometric Functions
- Selecting Procedures for Calculating Derivatives (Note: Use this lesson as review for differentiating functions.)
- Differentiating Functions Quiz

Week 3:

- Calculating Higher-Order Derivatives
- Personal Progress Check 3
- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Honors Calculus

Week 1:

- Section 3.1 The Chain Rules
- Section 3.1 Review
- Section 3.1 Quiz

Week 2:

- Section 3.2 Implicit Differentiation; Derivatives of the Inverse Trigonometric Functions
- Section 3.3 Derivatives of Logarithmic Functions

Week 3:

- Unit Review
- Unit Test

Unit 4: Contextual Applications of Differentiation

Content Area: **Math**
Course(s): **CALCULUS HONORS, AP CALC**
Time Period: **Marking Period 2**
Length: **3 Weeks**
Status: **Published**

Standards

Math Standards

MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Mathematical Practices

- 1.D: Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.
- 1.E: Apply appropriate mathematical rules or procedures, with and without technology.
- 1.F: Explain how an approximated value relates to the actual value.
- 2.A: Identify common underlying structures in problems involving different contextual situations. 2.B Identify mathematical.

- 3.D: Apply an appropriate mathematical definition, theorem, or test.
- 3.F: Explain the meaning of mathematical solutions in context.

Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

Transfer Goals

Transfer Goals

Unit 4 begins by developing understanding of average and instantaneous rates of change in problems involving motion. The unit then identifies differentiation as a common underlying structure on which to build understanding of change in a variety of contexts.

Concepts

Essential Questions

- How are problems about position, velocity, and acceleration of a particle in motion over time structurally similar to problems about the volume of a rising balloon over an interval of heights, the population of London over the 14th century, or the metabolism of a dose of medicine over time?
- Since certain indeterminate forms seem to actually approach a limit, how can we determine that limit, provided it exists?

Understandings

- Derivatives allow us to solve real-world problems involving rates of change.

- L’Hospital’s Rule allows us to determine the limits of some indeterminate forms.

Critical Knowledge and Skills

Knowledge

Students will know:

- The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
- The derivative can be used to express information about rates of change in applied contexts.
- The unit for $f'(x)$ is the unit for f divided by the unit for x.
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
- The derivative can be used to solve problems involving rates of change in applied contexts.
- The chain rule is the basis for differentiating variables in a related rates problem with respect to the same independent variable.
- Other differentiation rules, such as the product rule and the quotient rule, may also be necessary to differentiate all variables with respect to the same independent variable.
- The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
- The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
- For a tangent line approximation, the function’s behavior near the point of tangency may determine whether a tangent line value is an underestimate or an overestimate of the corresponding function value.
- When the ratio of two functions tends to 0/0 or ∞/∞ in the limit, such forms are said to be indeterminate.
- Limits of the indeterminate forms 0/0 or ∞/∞ may be evaluated using L’Hospital’s Rule.

Skills

Students will be able to:

- Interpret the meaning of a derivative in context.
- Calculate rates of change in applied contexts.
- Interpret rates of change in applied contexts.
- Calculate related rates in applied contexts.
- Interpret related rates in applied contexts.
- Approximate a value on a curve using the equation of a tangent line.

- Determine limits of functions that result in indeterminate forms.

Assessment and Resources

School Formative Assessment Plan (Other Evidence)

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

School Summative Assessment Plan

- Unit Assessment

Primary Resources

- Sullivan, Michael, and Kathleen Miranda. *Calculus: Early Transcendentals*. W.H. Freeman and Company, 2014.
- Finney, Ross L. *Calculus: Graphical, Numerical, Algebraic*. Pearson Prentice Hall, 2007.

Supplementary Resources

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMP
- YouTube
- AP Central
- Fostering Math Practices
- Rogawski, Jon. *Calculus. Early Transcendentals. Freeman, 2007.*

- Dietiker, Leslie, et al. *College Preparatory Mathematics: Calculus. CPM Educational Program, 2003.*

Technology Integration and Differentiated Instruction

Technology Integration

Google Products

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

One to One Student's Laptop

All students within the West Deptford School District are given a Chromebook, allowing for 21st century learning to occur within every lesson/topic.

Graphing Calculators

All students all required to utilize a graphing calculator, allowing for 21st century learning to occur within every lesson/topic.

Additional Support Videos

Videos can be assigned from PatrickJMP, Kahn Academy, YouTube, etc. to support each of the lessons within this topic.

Differentiated Instruction

Gifted Students (N.J.A.C.6A:8-3.1)

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

English Language Learners (N.J.A.C.6A:15)

- When discussing different ways of solving problems, focus on keywords and phrases. ELL Students might also benefit by using concrete objects to demonstrate different concepts.
- Create place cards or simple signs for students using the vocabulary words. They can use the signs as a reference throughout the lesson.
- Work with ELL Teacher to allow for all assignments to be completed with extra time.
- Pair ELL students with a student who is fluent in English.

At-Risk Students (N.J.A.C.6A:8-4.3c)

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

Special Education Students (N.J.A.C.6A:8-3.1)

- All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)
- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

Interdisciplinary Connections

ELA - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

SCIENCE - Students will analyze rates of change of 2D and 3D objects in terms of position, speed, height, distance, and angle of elevation.

SOCIAL STUDIES - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

WORLD LANGUAGES -

VISUAL/PERFORMING ARTS -

APPLIED TECHNOLOGY - Students will analyzie rates of change of electrical power and voltage. Students will make use of graphical interfaces such as Desmos and the TI graphing calculator suite.

BUSINESS EDUCATION - Students will analyze rates of change of cost, revenue, and profit.

GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- Interpreting the Meaning of the Derivative in Context
- Straight-Line Motion: Connecting Position, Velocity, and Acceleration
- Rates of Change in Applied Contexts Other Than Motion
- Rates of Change Review
- Rates of Change Quiz

Week 2:

- Introduction to Related Rates
- Solving Related Rates Problems
- Related Rates Review
- Related Rates Quiz

Week 3:

- Approximating Values of a Function using Local Linearity and Linearization
- Using L'Hospital's Rule for Determining Limits of Indeterminate Form
- Personal Progress Check 3
- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Honors Calculus

Week 1:

- Section 4.1 Related Rates
- Section 4.1 Review
- Section 4.1 Quiz

Note: Rates of Change in Context, Local Linearity, and L'Hopital's Rule are AP Calculus only. No learning plan necessary for Honors Calculus.

Unit 5: Analytical Applications of Differentiation

Content Area: **Math**
Course(s): **CALCULUS HONORS, AP CALC**
Time Period: **Marking Period 3**
Length: **4 Weeks**
Status: **Published**

Standards

Math Standards

MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.

MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MA.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Mathematical Practices

- 1.E: Apply appropriate mathematical rules or procedures, with and without technology.
- 2.A: Identify common underlying structures in problems involving different contextual situations.
- 2.D: Identify how mathematical characteristics or properties of functions are related in different representations.
- 2.E: Describe the relationships among different representations of functions and their derivatives.
- 3.D: Apply an appropriate mathematical definition, theorem, or test.
- 3.E: Provide reasons or rationales for solutions and conclusions.
- 3.F: Explain the meaning of mathematical solutions in context.

Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Transfer Goals

Transfer Goals

In this unit, the superficial details of contextual applications of differentiation are stripped away to focus on abstract structures and formal conclusions. Reasoning with definitions and theorems establishes that answers and conclusions are more than conjectures; they have been analytically determined.

Concepts

Essential Questions

- How might the Mean Value Theorem be used to justify a conclusion that you were speeding at some point on a certain stretch of highway, even without knowing the exact time you were speeding?
- What additional information is included in a sound mathematical argument about optimization that a simple description of an equivalent answer lacks?

Understandings

- Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.
- A function's derivative can be used to understand some behaviors of the function.

Critical Knowledge and Skills

Knowledge

Students will know:

- If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), then the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.
- If a function f is continuous over the interval (a, b), then the Extreme Value Theorem guarantees that f has at least one minimum value and at least one maximum value on (a, b).
- The first derivative of a function can provide information about the function and its graph, including intervals where the function is increasing or decreasing.
- The first derivative of a function can determine the location of relative (local) extrema of the function.
- Absolute (global) extrema of a function on a closed interval can only occur at critical points or at endpoints.
- The graph of a function is concave up (down) on an open interval if the function's derivative is increasing (decreasing) on that interval.
- The second derivative of a function provides information about the function and its graph, including intervals of upward or downward concavity
- The second derivative of a function may be used to locate points of inflection for the graph of the original function.
- The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum.
- When a continuous function has only one critical point on an interval on its domain and the critical point corresponds to a relative (local) extremum of the function on the interval, then that critical point

also corresponds to the absolute (global) extremum of the function on the interval.
- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- Graphical, numerical, and analytical information from f' and f'' can be used to predict and explain the behavior of f.
- Key features of the graphs of f, f', and f'' are related to one another.
- The derivative can be used to solve optimization problems; that is, finding a minimum or maximum value of a function on a given interval.
- Minimum and maximum values of a function take on specific meanings in applied contexts.
- A point on an implicit relation where the first derivative equals zero or does not exist is a critical point of the function.
- Applications of derivatives can be extended to implicitly defined functions.
- Second derivatives involving implicit differentiation may be relations of x, y, and dy/dx.

Skills

Students will be able to:

- Justify conclusions about functions by applying the Mean Value Theorem over an interval.
- Justify conclusions about functions by applying the Extreme Value Theorem.
- Justify conclusions about the behavior of a function based on the behavior of its derivatives.
- Calculate minimum and maximum values in applied contexts or analysis of functions.
- Interpret minimum and maximum values calculated in applied contexts.
- Determine critical points of implicit relations.
- Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives.

Assessment and Resources

School Formative Assessment Plan (Other Evidence)

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

School Summative Assessment Plan

- Unit Assessment

Primary Resources

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Technology Integration and Differentiated Instruction

Technology Integration

Google Products

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

One to One Student's Laptop

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Graphing Calculators

All students all required to utilize a graphing calculator, allowing for 21st century learning to occur within every lesson/topic.

Additional Support Videos

Videos can be assigned from PatrickJMP, Kahn Academy, YouTube, etc. to support each of the lessons within this topic.

Differentiated Instruction

Gifted Students (N.J.A.C.6A:8-3.1)

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

English Language Learners (N.J.A.C.6A:15)

- When discussing different ways of solving problems, focus on keywords and phrases. ELL Students might also benefit by using concrete objects to demonstrate different concepts.
- Create place cards or simple signs for students using the vocabulary words. They can use the signs as a reference throughout the lesson.
- Work with ELL Teacher to allow for all assignments to be completed with extra time.
- Pair ELL students with a student who is fluent in English.

At-Risk Students (N.J.A.C.6A:8-4.3c)

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

Special Education Students (N.J.A.C.6A:8-3.1)

- All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)
- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

Interdisciplinary Connections

ELA - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

SCIENCE -

SOCIAL STUDIES - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

WORLD LANGUAGES -

VISUAL/PERFORMING ARTS -

APPLIED TECHNOLOGY - Students will determine the optimal dimensions of 2D and 3D objects using graphical interfaces.

BUSINESS EDUCATION - Students will determine the optimal cost of fencing and can design. They will determine maximal revenue and profit with minimal cost.

GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- Using the Mean Value Theorem
- Extreme Value Theorem, Global Versus Local Extrema and Critical Points
- Determining Intervals on Which a Function is Increasing or Decreasing

Week 2:

- Using the First Derivative Test to Determine Relative (Local) Extrema
- Using the Candidates Test to Determine Absolute (Global) Extrema
- Determining Concavity of Functions over Their Domains

Week 3:

- Using the Second Derivative Test to Determine Extrema
- Sketching Graphs of Functions and Their Derivative
- Connecting a Function, Its First Derivative, and Its Second Derivative (Note: Use lesson to review graphs of functions and derivatives.)
- Graphs of Functions and Derivatives Quiz

Week 4:

- Introduction to Optimization Problems
- Solving Optimization Problems
- Personal Progress Check 5
- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Honors Calculus

Week 1:

- Section 4.2 Maximum and Minimum Values; Critical Numbers
- Section 4.2 Review
- Section 4.2 Quiz

Week 2:

- Section 4.4 Local Extrema and Concavity
- Section 4.4 Review
- Section 4.4 Quiz

Week 3:

- Section 4.7 Optimization
- Optimization Project

Week 4:

- Unit Review
- Unit Test

Unit 6: Integration and Accumulation of Change

Content Area: **Math**
Course(s): **CALCULUS HONORS, AP CALC**
Time Period: **Marking Period 3**
Length: **5 Weeks**
Status: **Published**

Standards

Math Standards

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – □, π + □, and 2π – □ in terms of their values for □, where □ is any real number.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.

Mathematical Practices

- 1.C: Identify an appropriate mathematical rule or procedure based on the classification of a given expression.
- 1.D: Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.
- 1.E: Apply appropriate mathematical rules or procedures, with and without technology.
- 1.F: Explain how an approximated value relates to the actual value.
- 2.C: Identify a re-expression of mathematical information presented in a given representation.
- 2.D: Identify how mathematical characteristics or properties of functions are related in different representations.
- 3.D: Apply an appropriate mathematical definition, theorem, or test.
- 4.B: Use appropriate units of measure.
- 4.C: Use appropriate mathematical symbols and notation.

Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or

	practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Transfer Goals

Transfer Goals

This unit establishes the relationship between differentiation and integration using the Fundamental Theorem of Calculus.

Concepts

Essential Questions

- Given information about a rate of population growth over time, how can we determine how much the population changed over a given interval of time?
- If compounding more often increases the amount in an account with a given rate of return and term, why doesn't compounding continuously result in an infinite account balance, all other things being equal?
- How is integrating to find areas related to differentiating to find slopes?

Understandings

- Definite integrals allow us to solve problems involving the accumulation of change over an interval.
- Definite integrals can be approximated using geometric and numerical methods.
- The Fundamental Theorem of Calculus connects differentiation and integration.
- Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.

Critical Knowledge and Skills

Knowledge

Students will know:

- The area of the region between the graph of a rate of change function and the x axis gives the accumulation of change.
- In some cases, accumulation of change can be evaluated by using geometry.
- If a rate of change is positive (negative) over an interval, then the accumulated change is positive (negative).
- The unit for the area of a region defined by rate of change is the unit for the rate of change multiplied by the unit for the independent variable.
- Definite integrals can be approximated for functions that are represented graphically, numerically, analytically, and verbally.
- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- Definite integrals can be approximated using numerical methods, with or without technology.
- Depending on the behavior of a function, it may be possible to determine whether an approximation for a definite integral is an underestimate or overestimate for the value of the definite integral.
- The limit of an approximating Riemann sum can be interpreted as a definite integral.
- A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- The definite integral of a continuous function f over the interval $[a, b]$ is the limit of Riemann sums as the widths of the subintervals approach 0.
- A definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
- The definite integral can be used to define new functions.
- If f is a continuous function on an interval containing a, then $d/dx(\int f(t)dt) = f(x)$, where x is in the interval.
- Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int f(t)dt$.
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- The definition of the definite integral may be extended to functions with removable or jump discontinuities.
- An antiderivative of a function f is a function g whose derivative is f.
- If a function f is continuous on an interval containing a, the function defined by $F(x)= \int f(t)dt$ is an antiderivative of f for x in the interval.
- If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int f(x)dx = F(b) - F(a)$.
- $\int f(x)dx$ is an indefinite integral of the function f and can be expressed as $\int f(x)dx = F(x) + C$, where $F'(x) = f(x)$ and C is any constant.
- Differentiation rules provide the foundation for finding antiderivatives.
- Many functions do not have closed-form antiderivatives.
- Substitution of variables is a technique for finding antiderivatives.
- For a definite integral, substitution of variables requires corresponding changes to the limits of integration.

- Techniques for finding antiderivatives include rearrangements into equivalent forms, such as long division and completing the square.

Skills

Students will be able to:

- Interpret the meaning of areas associated with the graph of a rate of change in context.
- Approximate a definite integral using geometric and numerical methods.
- Interpret the limiting case of the Riemann sum as a definite integral.
- Represent the limiting case of the Riemann sum as a definite integral.
- Represent accumulation functions using definite integrals.
- Calculate a definite integral using areas and properties of definite integrals.
- Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.
- Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.
- For integrands requiring substitution or rearrangements into equivalent forms:
    1. Determine indefinite integrals.
    2. Evaluate definite integrals.

Assessment and Resources

School Formative Assessment Plan (Other Evidence)

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

School Summative Assessment Plan

- Unit Assessment

Primary Resources

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Technology Integration and Differentiated Instruction

Technology Integration

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SCIENCE - Students will analyze the distance, velocity, and acceleration of objects.

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WORLD LANGUAGES -

VISUAL/PERFORMING ARTS -

APPLIED TECHNOLOGY - Students will make use of graphical interfaces such as Desmos and the TI graphing calculator suite.

BUSINESS EDUCATION -

GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- Exploring Accumulations of Change
- Approximating Areas with Riemann Sums

Week 2:

- Riemann Sums, Summation Notation, and Definite Integral Notation
- The Fundamental Theorem of Calculus and Accumulation Functions (Note: Use lesson to review approximating areas.)
- Approximating Areas Quiz

Week 3:

- Interpreting the Behavior of Accumulation Functions Involving Area
- Applying Properties of Definite Integrals
- The Fundamental Theorem of Calculus and Definite Integrals

Week 4:

- Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation
- Evaluating Integrals Review
- Evaluating Integrals Quiz

Week 5:

- Integrating Using Substitution
- Selecting Techniques for Antidifferentiation
- Personal Progress Check 6
- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Honors Calculus

Week 1:

- Section 4.8 Antiderivatives; Differential Equations
- Section 5.2 The Definite Integral

Week 2:

- Section 5.4 Properties of the Definite Integral
- Antiverivative and Definite Integral Review
- Antiverivative and Definite Integral Quiz

Week 3:

- Section 5.6 Method of Substitution

Week 4:

- Unit Review
- Unit Test

Unit 7: Differential Equations

Content Area: **Math**
Course(s): **AP CALC**
Time Period: **Marking Period 4**
Length: **2 Weeks**
Status: **Published**

Standards

Math Standards

MA.F-BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Mathematical Practices

- 1.E: Apply appropriate mathematical rules or procedures, with and without technology.
- 2.C: Identify a re-expression of mathematical information presented in a given representation.
- 3.G: Confirm that solutions are accurate and appropriate.
- 4.D: Use appropriate graphing techniques.

Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Transfer Goals

Transfer Goals

In this unit, students will learn to set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration.

Concepts

Essential Questions

- How can we derive a model for the number of computers, C, infected by a virus, given a model for how fast the computers are being infected, dC/dt, at a particular time?

Understandings

- Solving differential equations allows us to determine functions and develop models.

Critical Knowledge and Skills

Knowledge

Students will know:

- Differential equations relate a function of an independent variable and the function's derivatives.
- Derivatives can be used to verify that a function is a solution to a given differential equation.
- There may be infinitely many general solutions to a differential equation.
- A slope field is a graphical representation of a differential equation on a finite set of points in the plane.
- Slope fields provide information about the behavior of solutions to first-order differential equations.
- Solutions to differential equations are functions or families of functions.
- Some differential equations can be solved by separation of variables.
- Antidifferentiation can be used to find general solutions to differential equations.

- A general solution may describe infinitely many solutions to a differential equation. There is only one particular solution passing through a given point.
- The function F defined by $F(x) = y_0 + \int f(t)dt$ is a particular solution to the differential equation $dy/dx = f(x)$, satisfying $F(a) = y_0$.
- Solutions to differential equations may be subject to domain restrictions.
- Specific applications of finding general and particular solutions to differential equations include motion along a line and exponential growth and decay
- The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $dy/dt = ky$.
- The exponential growth and decay model, $dy/dt = ky$, with initial condition $y = y_0$ when $t = 0$, has solutions of the form $y = y_0e^{kt}$.

Skills

Students will be able to:

- Interpret verbal statements of problems as differential equations involving a derivative expression.
- Verify solutions to differential equations.
- Estimate solutions to differential equations.
- Determine general solutions to differential equations.
- Determine particular solutions to differential equations.
- Interpret the meaning of a differential equation and its variables in context.
- Determine general and particular solutions for problems involving differential equations in context.

Assessment and Resources

School Formative Assessment Plan (Other Evidence)

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

School Summative Assessment Plan

- Unit Assessment

Primary Resources

- Sullivan, Michael, and Kathleen Miranda. *Calculus: Early Transcendentals*. W.H. Freeman and Company, 2014.
- Finney, Ross L. *Calculus: Graphical, Numerical, Algebraic*. Pearson Prentice Hall, 2007.

Supplementary Resources

- IXL
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Graphing Calculators

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Additional Support Videos

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Differentiated Instruction

Gifted Students (N.J.A.C.6A:8-3.1)

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

English Language Learners (N.J.A.C.6A:15)

- When discussing different ways of solving problems, focus on keywords and phrases. ELL Students might also benefit by using concrete objects to demonstrate different concepts.
- Create place cards or simple signs for students using the vocabulary words. They can use the signs as a reference throughout the lesson.
- Work with ELL Teacher to allow for all assignments to be completed with extra time.
- Pair ELL students with a student who is fluent in English.

At-Risk Students (N.J.A.C.6A:8-4.3c)

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Special Education Students (N.J.A.C.6A:8-3.1)

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Interdisciplinary Connections

ELA - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

SCIENCE -

SOCIAL STUDIES - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

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BUSINESS EDUCATION -

GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- Modeling Situations with Differential Equations
- Verifying Solutions for Differential Equations
- Sketching Slope Fields
- Reasoning Using Slope Fields

Week 2:

- Finding General Solutions Using Separation of Variables
- Finding Particular Solutions Using Initial Conditions and Separation of Variables
- Exponential Models with Differential Equations
- Personal Progress Check 7

- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Note: This unit is for AP Calculus only; No learning plan necessary for Honors Calculus.

Unit 8: Applications of Integration

Content Area: **Math**
Course(s): **CALCULUS HONORS, AP CALC**
Time Period: **Marking Period 4**
Length: **5 Weeks**
Status: **Published**

Standards

Math Standards

MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Mathematical Practices

- 1.D: Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.
- 1.E: Apply appropriate mathematical rules or procedures, with and without technology.
- 2.B: Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
- 2.D: Identify how mathematical characteristics or properties of functions are related in different representations.
- 3.D: Apply an appropriate mathematical definition, theorem, or test.
- 4.C: Use appropriate mathematical symbols and notation.
- 4.E: Apply appropriate rounding procedures.

Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.,

	1.1.12prof.CR3a).
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TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
TECH.9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

Transfer Goals

Transfer Goals

In this unit, students will learn how to find the average value of a function, model particle motion and net change, and determine areas, and volumes defined by the graphs of functions. Emphasis should be placed on developing an understanding of integration that can be transferred across these and many other applications.

Concepts

Essential Questions

- How is finding the number of visitors to a museum over an interval of time based on information about the rate of entry similar to finding the area of a region between a curve and the x-axis?

Understandings

- Definite integrals allow us to solve problems involving the accumulation of change over an interval.
- Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.

Critical Knowledge and Skills

Knowledge

Students will know:

- The average value of a continuous function f over an interval $[a, b]$ is $1/(a - b)\int f(x)dx$.
- For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
- A function defined as an integral represents an accumulation of a rate of change.
- The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- The definite integral can be used to express information about accumulation and net change in many applied contexts.
- Areas of regions in the plane can be calculated with definite integrals.
- Areas of regions in the plane can be calculated using functions of either x or y.
- Areas of certain regions in the plane may be calculated using a sum of two or more definite integrals or by evaluating a definite integral of the absolute value of the difference of two functions.
- Volumes of solids with square and rectangular cross sections can be found using definite integrals and the area formulas for these shapes.
- Volumes of solids with triangular cross sections can be found using definite integrals and the area formulas for these shapes.
- Volumes of solids with semicircular and other geometrically defined cross sections can be found using definite integrals and the area formulas for these shapes.
- Volumes of solids of revolution around the x- or y-axis may be found by using definite integrals with the disc method.
- Volumes of solids of revolution around any horizontal or vertical line in the plane may be found by using definite integrals with the disc method.
- Volumes of solids of revolution around the x- or y-axis whose cross sections are ring shaped may be found using definite integrals with the washer method.
- Volumes of solids of revolution around any horizontal or vertical line whose cross sections are ring shaped may be found using definite integrals with the washer method.

Skills

Students will be able to:

- Determine the average value of a function using definite integrals.
- Determine values for positions and rates of change using definite integrals in problems involving rectilinear motion.
- Interpret the meaning of a definite integral in accumulation problems.
- Determine net change using definite integrals in applied contexts.
- Calculate areas in the plane using the definite integral.
- Calculate volumes of solids with known cross sections using definite integrals.
- Calculate volumes of solids of revolution using definite integrals.

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GLOBAL AWARENESS -

Learning Plan / Pacing Guide

AP Calculus

Week 1:

- Finding the Average Value of a Function on an Interval
- Connecting Position, Velocity, and Acceleration of Functions Using Integrals
- Using Accumulation Functions and Definite Integrals in Applied Contexts

Week 2:

- Finding the Area Between Curves Expressed as Functions of x
- Finding the Area Between Curves Expressed as Functions of y
- Finding the Area Between Curves That Intersect at More Than Two Points

Week 3:

- Definite Integrals Applied in Context and Area Between Curves Review
- Definite Integrals Applied in Context and Area Between Curves Quiz
- Volumes with Cross Sections: Squares and Rectangles
- Volumes with Cross Sections: Triangles and Semicircles

Week 4:

- Volume with Disc Method: Revolving Around the x - or y - axis
- Volume with Disc Method: Revolving Around Other Axis
- Volume with Washer Method: Revolving Around the x - or y - axis
- Volume with Washer Method: Revolving Around Other Axis

Week 5:

- Volumes with Cross Sections, Disc Method, and Washer Method Review
- Volumes with Cross Sections, Disc Method, and Washer Method Quiz (Note: Omit quiz if cylindrical shells is not covered.)
- Volume with Cylindrical Shells Method: Revolving Around the x - or y - axis (Note: Cover cylindrical shells if time permits.)
- Volume with Cylindrical Shells Method: Revolving Around Other Axis
- Personal Progress Check 8
- Unit Assessment (Note: Administer assessment one week after assigning personal progress check.)

Honors Calculus

Week 1:

- Section 6.1 Area Between Graphs
- Section 6.1 Review

- Section 6.1 Quiz

Week 2:

- Section 6.2 Volume of a Solid of Revolution: Disks and Washers
- Section 6.2 Review
- Section 6.2 Quiz

Week 3:

- Section 6.3 Volume of a Solid of Revolution: Cylindrical Shells

Week 4:

- Unit Review
- Unit Test